

## REFERENCES

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### DETERMINATION OF THE SEPARATION EFFECT IN A STIRRED VESSEL WITH A SUSPENSION BY A DYNAMIC METHOD

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In a previous paper<sup>1</sup> equations were derived for the value of the separation coefficient (ratio of the solid concentration in the outlet to the average solids concentration in the vessel) for the isokinetic withdrawal of a suspension from a stirred vessel. These are

$$\varphi = [1 - 2X/D_e(u_e/u_i)^{1/2}]^2, \quad u_e/u_i > 1 \quad (1)$$

where  $X$  is determined from

$$X/(u_e/u_i)^{1/2} = (1/k) [(q_p - q_k) D_e/q_k] \ln \{ [k q_k/D_e(q_p - q_k)] \cdot \\ \cdot \{ (D_e/2) [(u_e/u_i)^{1/2} - 1] + X[(u_i/u_e)^{1/2} - 1] \} + 1 \} \quad (2)$$

and

$$\varphi = [1 + 2Y/D_e(u_e/u_i)^{1/2}]^2, \quad (3)$$

where  $Y$  is determined from

$$Y/(u_e/u_i)^{1/2} = (1/k) [(q_p - q_k) D_e/q_k] \ln \{ [k q_k/D_e(q_p - q_k)] \cdot \\ \cdot \{ (D_e/2) [1 - u_e/u_i]^{1/2} + Y[(u_i/u_e)^{1/2} - 1] \} + 1 \}. \quad (4)$$

The constant  $k$  in Eq. (2) and (4) depends only on the geometry of the system. The aim of this work was to verify the validity of relations (1)–(4) and to determine the value of the constant for various physical systems and a given geometry. The experiments were performed with only one geometry of the system. The independent variables were: stirrer speed, volumetric flow rate

of suspension, density of liquid and solid particles and diameter of the solid particles. The range of values of the independent variables is shown in Table I.

TABLE I  
Range of Values of Independent Variables

$n$ rev $\text{min}^{-1}$	$F$ $\text{l min}^{-1}$	$d_p$ cm	$\rho_p$ $\text{g cm}^{-3}$	$w_0$ $\text{cm s}^{-1}$	$\rho_k$ $\text{g cm}^{-3}$
450–1300	0.5–4	0.018–0.09	1.018–2.665	0.55–15.75	0.78–1

## EXPERIMENTAL

### Equipment and Method

A drawing of the vessel used is shown on Fig. 1. The vessel was stirred by a standard six-blade turbine and there were 4 standard baffles at the wall. The outlet was placed in the same height as the stirrer blades and in the middle between two baffles. The vessel was full of liquid right up to the top cover, thus eliminating vortex formation.

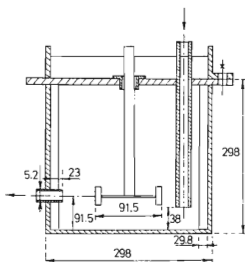


FIG. 1  
Schematic Drawing of Vessel Used for Experiments

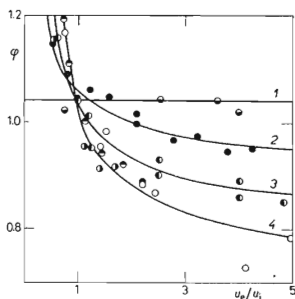


FIG. 2  
Measured Values of the Separation Coefficients vs Ratio  $u_e/u_i$

1 ● Polystyrene–water, 2 ● plexiglass–kerosene, 3 ● sugar–kerosene, 4 ○ glass 0.16–0.2 mm–water, ● glass 0.4–0.5 mm–water, ● glass 0.8–1 mm–water.

In order to eliminate the time required for obtaining steady state conditions and to use the minimum amount of classified solid particles, a dynamic method of measuring the separation coefficient at unsteady state conditions was chosen. A given weighed amount of solid material was at first introduced into the vessel. At time  $t = 0$  the suspension began to be continuously withdrawn from the system and pure liquid was continuously introduced so that the volumetric contents of the vessel remained constant with time. This causes a continuous decrease of the concentration of solid particles inside the vessel. Very small concentrations of the solid particles were employed so that it could be assumed that the separation coefficient was independent of the concentration. The volumetric concentration of the solid particles at the beginning of the measurements varied from 0.18 to 0.25%. We used higher stirrer speeds than Rushton<sup>2</sup> and therefore also assumed complete homogenisation of the solid particles inside the vessel.

The mass balance of the solid material in the vessel is

$$0 = Fc_e + V dc_i/dt. \quad (5)$$

We assume that the volumetric flow rate of the suspension  $F$  is equal to the volumetric flow rate of the liquid. Since  $\varphi = c_e/c_i$  by definition we obtain on substitution into Eq. (5)

$$dc_i/c_i = - (1/\bar{v}) \varphi dt. \quad (6)$$

Integrating Eq. (6) from  $t_1$  to  $t_2$  we obtain the final expression used for calculating the separation coefficient

$$\ln(c_{i2}/c_{i1}) = - (1/\bar{v}) \varphi(t_2 - t_1). \quad (7)$$

The average concentration of the solid particles inside the vessel was determined by weighing the solid material which left the vessel from the beginning of the measurement and subtracting this from the original amount charged.

The density of the liquid and solid phase was determined by means of a pycnometer. For each solid particle fraction and liquid five measurements were made. The average values are presented in Table II.

TABLE II  
Physical Properties of the Measured Systems

Curve	Solid	$\rho_p$ g/cm <sup>3</sup>	$d_p$ mm	Liquid	$\rho_p/\rho_k$	$w_0$ cm/s
1	polystyrene	1.03	0.755	water	1.03	0.69
2	plexiglass	1.166	0.55	kerosene	1.491	1.86
3	sugar	1.586	0.605	kerosene	2.028	5.7
4	glass	2.655	0.18	water	2.655	1.98
4	glass	2.655	0.45	water	2.655	8
4	glass	2.655	0.9	water	2.655	15.75

The free-fall velocity of the classified solid particles in the liquid used for measurements was determined in a pipe of diameter 150 mm and length 2000 mm for a distance of 1 m at a temperature of 20°C. The average values of 50 measurements are presented in Table II.

The diameter of the solid particles was calculated from the measured free-fall velocity. The particles were classified on sieves and polystyrene particles were separated according to their density by means of a concentrated NaCl solution. Two fractions of 1.015–1.025 g/cm<sup>3</sup> and 1.025–1.045 g/cm<sup>3</sup> were obtained.

The stirrer speed was measured by a calibrated voltmeter and the calibration was made with a hand speedometer with an accuracy of  $\pm 2\%$ . The isokinetic velocity of the liquid inside the vessel in the vicinity of the outlet was calculated according to Rushton by the relation

$$u_i = Bnd/r. \quad (8)$$

For the constant  $B$  a value of 1.12 was chosen by interpolation from the values given by Rushton to correspond to our geometry.

#### RESULTS AND DISCUSSION

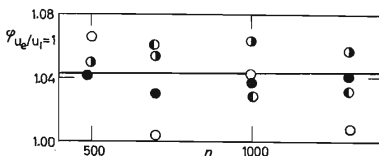
The measured values of the separation coefficient *vs.* the ratio of velocities  $u_c/u_i$  are shown in Fig. 2. From this figure it is evident that in agreement with Eqs (1) and (3) a diminishing value of the density difference causes a decrease of the effect of  $u_c/u_i$  on the separation coefficient and in the case of curve 1 the effect is negligible. In agreement with theory the particle diameter also has a small effect on the value of the separation coefficient because all values for the system glass-water for a wide range of particle diameters lie on the same curve. From the figure it is further apparent that for velocity ratios equal to one the measured values of the separation coefficient are not equal to unity. The straight line  $\varphi = 1$  passes through the measured curves at different values of  $u_c/u_i$ . This indicates that it cannot be caused by a faulty determination of the isokinetic velocity. A possible explanation is that the solid particle concentration is not the same at all points inside the vessel and the calculated average concentration is not the same as the concentration in the vicinity of the outlet. In order to obtain the actual value of the separation coefficient the measured values have to be multiplied by a correction factor according to the relation

$$\varphi_s = \varphi(c_i/c_L). \quad (9)$$

For a velocity ratio equal to unity the correction factor  $c_i/c_L$  can be determined from the equation

$$\varphi_{(u_c/u_i=1)}(c_i/c_L) = 1. \quad (10)$$

FIG. 3  
Values of  $\varphi_{(u_c/u_i=1)}$  vs Stirrer Speed  
Experimental points have the same meaning as in Fig. 2.



Because the local concentration  $c_L$  can be for the given geometry in general a function of the stirrer speed, suspension flow rate and physical properties of the system we determined how the value of  $\varphi_{(u_e/u_i=1)}$  depended on these variables. On Fig. 3 is shown how the separation coefficient depends on the stirrer speed for a constant ratio  $(u_e/u_i) = 1$  for the systems employed in our measurements. From the figure it is apparent that within accuracy of the data the separation coefficient can be considered as being constant. The actual values of the separation coefficient were then determined by the relation

$$\varphi_s = \varphi/1.043. \quad (11)$$

The value 1.043 is the arithmetic average of the values shown on Fig. 3 ( $\sigma_r = 0.056$ ). The actual separation coefficients thus calculated were compared with the theoretical values determined from Eqs (1) and (3). By the method of least squares it was found that for all physical systems and for the system geometry used the best value of  $k$  was 123.5. The root mean square relative deviation between the calculated and measured values of the separation coefficient was 0.74% which proves the validity of Eqs (1) and (3) for very dilute suspensions.

#### LIST OF SYMBOLS

- $B$  constant in Eq. (8)  
 $c_e$  concentration of the solid particles at the outlet of the vessel ( $ML^{-3}$ )  
 $c_i$  average concentration of solids inside the vessel ( $ML^{-3}$ )  
 $c_L$  local concentration of solids inside the vessel in the vicinity of the outlet ( $ML^{-3}$ )  
 $D_e$  diameter of the outlet (L)  
 $d$  stirrer diameter (L)  
 $d_p$  diameter of the solid particles (L)  
 $F$  volumetric flow rate of the suspension ( $L^3 t^{-1}$ )  
 $k$  constant  
 $n$  stirrer speed ( $t^{-1}$ )  
 $r$  radial distance of outlet from stirrer axis (L)  
 $t$  time (t)  
 $\bar{t}$  mean residence time of suspension (t)  
 $u_e$  linear velocity of the suspension in the outlet ( $Lt^{-1}$ )  
 $u_i$  linear velocity of the suspension inside the vessel in the vicinity of the outlet ( $Lt^{-1}$ )  
 $V$  vessel volume ( $L^3$ )  
 $w_0$  free-fall velocity ( $Lt^{-1}$ )  
 $X$  value defined by Eq. (1) (L)  
 $Y$  value defined by Eq. (3) (L)  
 $\varphi$  separation coefficient  $c_e/c_i$   
 $\varphi_s$  corrected value of the actual separation coefficient  
 $\sigma_r$  root mean square relative deviation  
 $\rho_k$  liquid density ( $ML^{-3}$ )  
 $\rho_p$  density of the solid particles ( $ML^{-3}$ )

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